GCSE Maths – Geometry and Measures

Volume of 3D Shapes

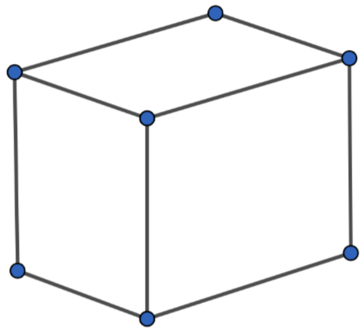
Notes

WORKSHEET

3D Shapes

Unlike 2D shapes, 3D shapes are not flat. Instead, they have **three dimensions**: length, width, and height.

This is a cuboid. Other types of 3D shapes include spheres, cones, cubes, pyramids, and cylinders.



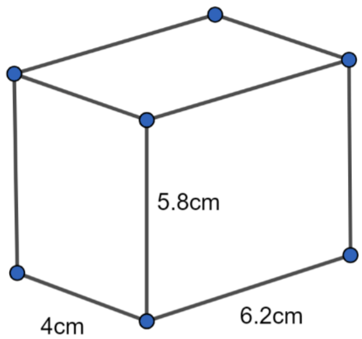
Volume refers to **how much space** is contained **within a 3D shape**. Volume is one dimension larger than area (which is the amount of space contained within 2D shapes). We must ensure we use the correct **units** for volume: units^3, such as cm^3.

Volume of 3D Shapes

Volume of cuboids

$$Volume\ of\ cuboid = Length \times Width \times Height$$

For example, we can find the area of the following cuboid:



We have been given the length (6.2 cm), the width (4 cm) and the height (5.8 cm). Multiplying these together will give us the volume:

$$Volume = 6.2 \times 4 \times 5.8 = 143.84\ \text{cm}^3$$

A **cube** is a type of **cuboid** where the length, width, and height are all the same. Therefore, if we know the volume of a cube, we can calculate the cube root of the volume ($\sqrt[3]{volume}$) to find the length of each side.

Example: A cube has a volume of $125\ \text{cm}^3$. What is the length of each side of the cube?

For a cuboid,

$$Volume = Length \times Width \times Height = lwh$$

In a cube, the length, width and height are all the same, so the formula becomes

$$Volume = l^3$$

Given, that the volume is $125\ \text{cm}^3$, we have the equation:

$$l^3 = 125$$

Take the cube root of each side:

$$\sqrt[3]{l^3} = \sqrt[3]{125}$$
$$l = 5\ \text{cm}$$

Each side is **5 cm** *in length.*

Volume of prisms

A prism is 3D shape that has **two identical faces** opposite one another. All its faces are flat.

To calculate the volume of a prism, we first need to work out the **area** of the **cross-section**. The cross-section face will typically be a polygon you are familiar with finding the area of, like a triangle. We then find the volume by multiplying the cross-section area by the length of the prism:

$$Volume\ of\ prism = Cross\ section\ area \times Length$$

For example, consider the following prism:

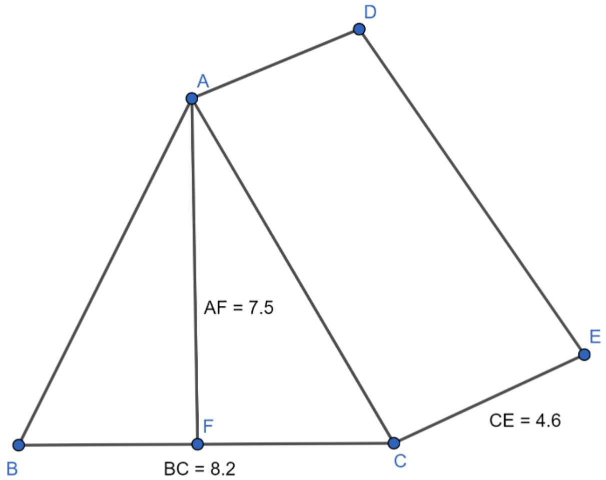
The triangular face ABC at the front is the cross-section. First, calculate this area of this face using the formula for the area of a triangle:

$$Area = \frac{Base \times Height}{2} = \frac{8.2 \times 7.5}{2}$$

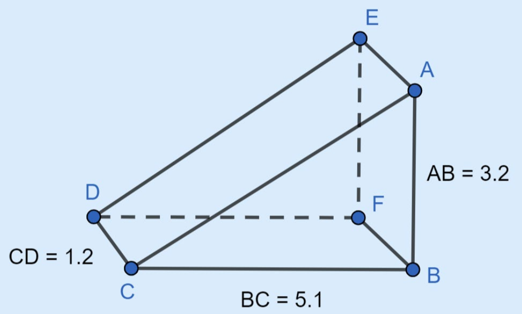
$$= 30.75 \text{ units}^2$$

Now, ***multiply*** *the cross-sectional area by the* ***length*** *of the prism.*

$$Volume = 30.75 \times 4.6 = \mathbf{141.45\ units^3}$$



Example: Find the volume of the following prism.



1. Find the area of the cross-section triangular face ABC.

$$Area = \frac{Base \times Height}{2} = \frac{5.1 \times 3.2}{2} = 8.16 \text{ units}^2$$

2. Multiply the cross-sectional area by the length of the prism to find the volume.

$$Volume = Cross\ section\ area \times Length$$

$$= 8.16 \times 1.2 = \mathbf{9.79\ units^3}\ (2\text{ d.p.})$$

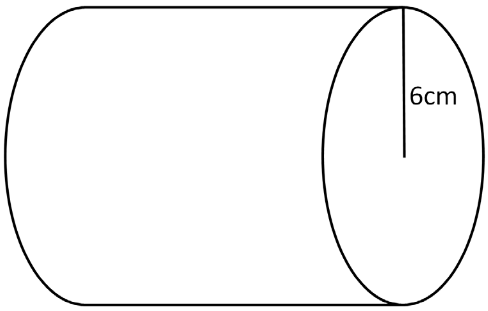
Volume of cylinders

Cylinders are 3D shapes that look like **tubes**.

To calculate the volume of a cylinder, we use the same approach as we did for prisms: find the **cross-sectional area** (a circular face in cylinders), and **multiply by the height** of the cylinder.

$$Volume\ of\ cone = \pi r^2 \times h$$

For example, consider the following cylinder:



First, we need to calculate the area of the circular cross-sectional face.
Use the formula for the area of a circle:

$$Area = \pi \times r^2 = \pi \times 6^2 = 113.1\ \text{cm}^2\ (2\ \text{d.p.})$$

Now multiply the cross-sectional area by the length of the cylinder:

$$Volume = 113.1 \times 20 = \mathbf{2261.9\ cm^3}\ (2\ \text{d.p.})$$

Example: A cylinder has a face with diameter 3 cm and the length of the cylinder is 9 cm. Calculate its volume.

1. Calculate the cross-sectional area using the radius of the circular face.

   We are given the diameter so we must remember to halve the diameter to find the radius:

   $$Radius = Diameter \div 2 = 3 \div 2 = 1.5\ \text{cm}$$

   So,

   $$Area = \pi \times 1.5^2 = 7.07\ \text{cm}^2\ (2\ \text{d.p.})$$

2. Multiply the cross-sectional area by the length to find the volume.

   $$Volume = 7.07 \times 9 = \mathbf{63.63\ cm^3}\ (2\ \text{d.p.})$$

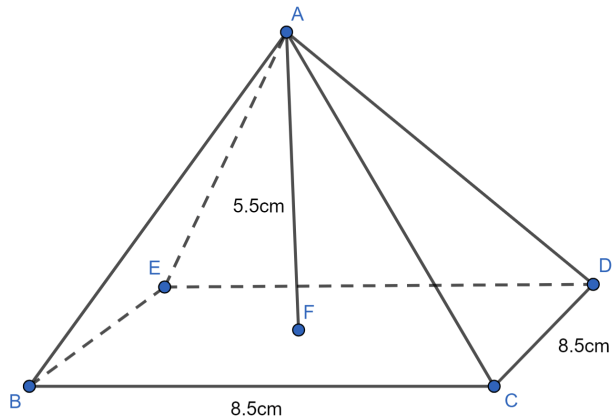
Volume of pyramids

Pyramids are another type of 3D shape. Pyramids can have different 2D shapes as their **base** – triangles, squares or rectangles. The formula for calculating the volume of a pyramid is:

$$Volume = \frac{1}{3} \times Perpendicular\ height \times Area\ of\ base$$

The perpendicular height is the **direct height** from the **base** to the **tip** of the pyramid, not the length of the sloped sides.

For example, let's calculate the volume of the following square-based pyramid:



First, we need to calculate the area of the base. This is a square-based pyramid, so to calculate the area, we simply multiply the length by the width.

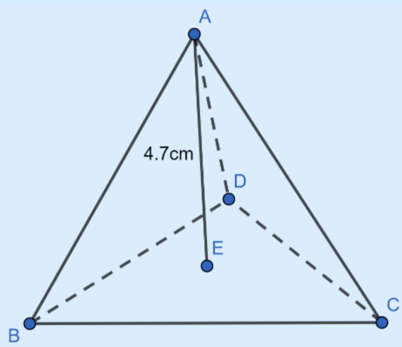
$$Area\ of\ base = 8.5 \times 8.5 = 72.25\ \text{cm}^2$$

Now we can use this value in the formula to find the volume. We have been given the perpendicular height of the pyramid as 5.5 cm *so we get:*

$$Volume = \frac{1}{3} \times 5.5 \times 72.25 = \mathbf{132.46\ cm^3}$$

Example: The area of the triangular base of this pyramid is $45\ \text{cm}^2$.

Find the volume of this pyramid.



$$Volume = \frac{1}{3} \times Perpendicular\ height \times Area\ of\ base$$

$$Volume = \frac{1}{3} \times 4.7 \times 45 = \mathbf{70.5\ cm^3}$$

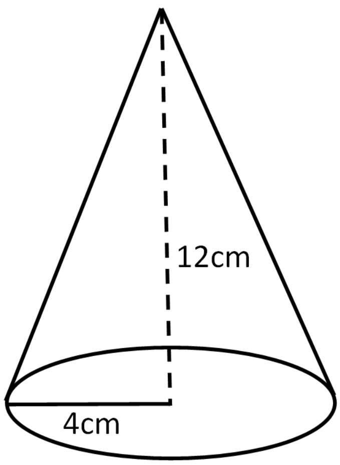
Volume of cones

The **formula** for the volume of a **cone** is:

$$Volume = \frac{1}{3}\pi \times r^2 \times h$$

You might notice that this formula is very similar to the way we work out the volume of a cylinder. This is because the volume of a cone is **$\frac{1}{3}$ the volume of a cylinder**.

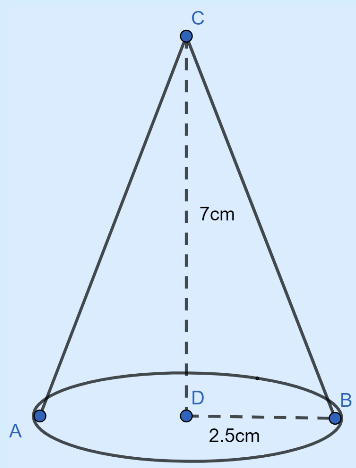
Let's use the cone below as an example:



Use the formula to calculate the volume:

$$Volume = \frac{1}{3} \times \pi \times 4^2 \times 12 = \mathbf{201.06\ cm^3}$$

Example: Calculate the volume of the following cone.



Use the formula for the volume of a cone, and substitute in the given values:

$$Volume = \frac{1}{3}\pi \times r^2 \times h = \frac{1}{3} \times \pi \times 2.5^2 \times 7 = \mathbf{45.81\ cm^3}$$

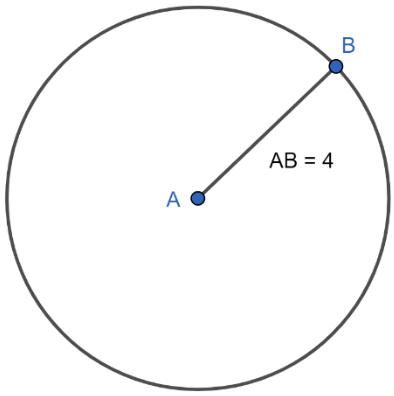
Volume of spheres

The formula for the volume of a sphere is:

$$Volume = \frac{4}{3} \times \pi \times r^3$$

All we need to know is the **radius** of the sphere.

To find the volume of the following sphere, we use the formula:



$$Volume = \frac{4}{3} \times \pi \times r^3$$

$$= \frac{4}{3} \times \pi \times 4^3 = \mathbf{268.08\ units^3}\ (2\ \text{d. p.})$$

Example: Find the volume of a sphere with a diameter of 7 cm.

1. Find the radius of the sphere.

   To find the radius, we need to halve the diameter:

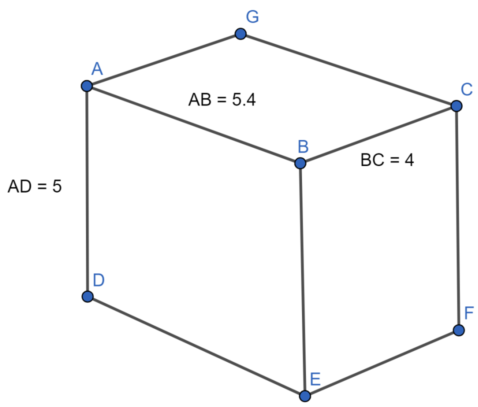
   $$Radius = Diameter \div 2 = 7 \div 2 = 3.5\ \text{cm}$$

2. *Use the formula to calculate volume of a sphere.*

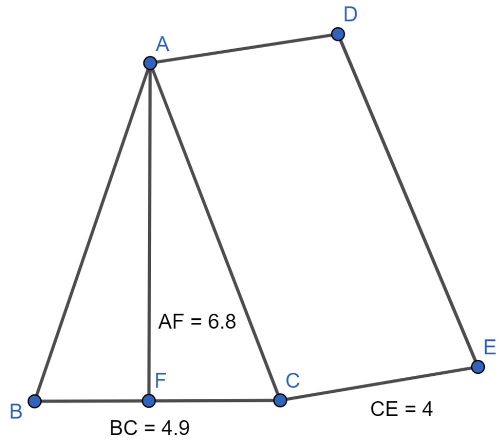
   $$Volume = \frac{4}{3} \times \pi \times r^3 = \frac{4}{3} \times \pi \times 3.5^3 = \mathbf{179.59\ cm^3}$$

Volumes of 3D Shapes – Practice Questions

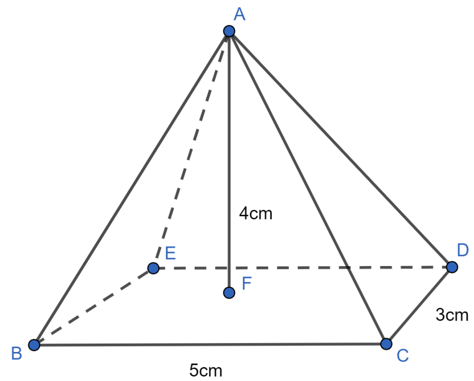
1. Calculate the volume of the following cuboid:



2. Calculate the volume of the following prism:



3. Calculate the volume of the following pyramid:



Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.